SAFE HANDS & IIT-ian's PACE

LEAP TEST# 08 (JEE) ANS KEY Dt. 20-12-2023

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	D	31	A	61	C
2	C	32	A	62	A
3	D	33	A	63	A
4	D	34	A	64	B
5	C	35	B	65	A
6	A	36	C	66	A
7	B	37	B	67	D
8	C	38	C	68	C
9	B	39	C	69	C
10	A	40	D	70	B
11	D	41	A	71	C
12	B	42	A	72	B
13	B	43	D	73	B
14	D	44	A	74	D
15	B	45	A	75	A
16	C	46	A	76	B
17	C	47	D	77	A
18	D	48	B	78	B
19	C	49	A	79	C
20	A	50	A	80	B
21	944.44	51	8	81	5
22	4	52	6	82	3
23	0.96	53	6	83	2
24	0.45	54	0	84	5
25	1.54	55	6	85	8

See Maths solutions on next page.....

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Answers: LT 08 Mathematics JEE

61)	c	62)	a	63)	a	64)	b	77)	a	78)	b	79)	c	80)	b
65)	a	66)	a	67)	d	68)	c	81)	5	82)	3	83)	2	84)	5
69)	c	70)	b	71)	c	72)	b	85)	8						
73)	b	74)	d	75)	a	76)	b								

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Solution to

LT 08 Mathematics JEE

Single Correct Answer Type

61 **(c)**

Since, $y^2 = P(x)$

On differentiating both sides, we get

$$2yy_1 = P'(x),$$

Again, differentiating, we get

$$2yy_2 + 2y_1^2 = P''(x)$$

$\Rightarrow$ $$2y^3y_2 + 2y^2y_1^2 = y^2P''(x)$$

$\Rightarrow$ $$2y^3y_2 = y^2P''(x) - 2(yy_1)^2$$

$\Rightarrow$ $$2y^3y_2 = P(x)\cdot P''(x) - \frac{\{P'(x)\}^2}{2}$$

Again, differentiating, we get

$$2\frac{d}{dx}(y^3y_2) = P'(x)\cdot P''(x) + P(x)\cdot P'''(x) - \frac{2P'(x)\cdot P''(x)}{2}$$

$\Rightarrow$ $$2\frac{d}{dx}(y^3y_2) = P(x)\cdot P'''(x)$$

$\Rightarrow$ $$2\frac{d}{dx}\left(y^3\cdot\frac{d^2y}{dx^2}\right) = P(x)\cdot P'''(x)$$

62 **(a)**

We have

$$y^{1/m} = (x + \sqrt{1+x^2})$$

$$\Rightarrow y = \left(x + \sqrt{1+x^2}\right)^m$$

$$\Rightarrow \frac{dy}{dx} = m\left(x + \sqrt{1+x^2}\right)^{m-1}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$$

$$= m\frac{\left(x+\sqrt{1+x^2}\right)^m}{\sqrt{1+x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{my}{\sqrt{1+x^2}}$$

$$\Rightarrow y_1^2(1+x^2) = m^2y^2$$

$$\Rightarrow 2y_1y_2(1+x^2) + 2xy_1^2 = 2m^2yy_1$$

$$\Rightarrow y_2(1+x^2) + xy_1 = m^2y$$

63 **(a)**

Given $f = f' + f'' + f''' + \cdots\infty$

$\Rightarrow f' = f'' + f''' + f'''' + \cdots\infty$

$\Rightarrow f - f' = f'$

$\Rightarrow f = 2f'$

Hence, $\frac{f'}{f} = 1/2 \Rightarrow \int\frac{f'}{f}dx = \int\frac{1}{2}dx$

$\Rightarrow \log f(x) = x/2 + c$

$\Rightarrow f(x) = e^{x/2+c}$

Also, $f(0) = 1 \Rightarrow c = 0 \Rightarrow f(x) = e^{x/2}$

64 **(b)**

Since, $f''(x) = -f(x)$

$\Rightarrow$ $\frac{d}{dx}\{f'(x)\} = -f(x)$

$\Rightarrow$ $g'(x) = -f(x)$ [$\because$ $g(x) = f'(x)$, given] ...(i)

Also, $F(x) = \left\{f\left(\frac{x}{2}\right)\right\}^2 + \left\{g\left(\frac{x}{2}\right)\right\}^2$

$\Rightarrow$ $F'(x) = 2\left(f\left(\frac{x}{2}\right)\right)\cdot f'\left(\frac{x}{2}\right)\cdot\frac{1}{2}$

$+ 2\left(g\left(\frac{x}{2}\right)\right)\cdot g'\left(\frac{x}{2}\right)\cdot\frac{1}{2} = 0$

[from Eq. (i)]

$\therefore$ $F(x)$ is constant $\Rightarrow$ $F(10) = F(5) = 5$

65 **(a)**

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(x+\sqrt{x^2+a^2}\right)^n\right]$$

$$= n\left(x+\sqrt{x^2+a^2}\right)^{n-1}.\frac{d}{dx}\left(x+\sqrt{x^2+a^2}\right)$$

$$= n\left(x+\sqrt{x^2+a^2}\right)^{n-1}\left(\frac{\sqrt{x^2+a^2}+a^2}{\sqrt{x^2+a^2}}\right)$$

$$= \frac{n\left(x+\sqrt{x^2+a^2}\right)^n}{\sqrt{x^2+a^2}}$$

$$= \frac{ny}{\sqrt{x^2+a^2}}$$

66 **(a)**

Since, $f(x) = e^{g(x)}$ $\Rightarrow$ $e^{g\,(x+1)} = f(x+1) = xf(x) = xe^{g\,(x)}$

and $g\,(x+1) = \log x + g\,(x)$

i.e. $g\,(x+1) - g\,(x) = \log x$ ...(i)

Replacing x by $-\frac{1}{2}$, we get

$g\left(x+\frac{1}{2}\right) - g\left(x-\frac{1}{2}\right) = \log\left(x-\frac{1}{2}\right) = \log(2x-1) - \log 2$

$\therefore$ $g''\left(x+\frac{1}{2}\right) - g''\left(x-\frac{1}{2}\right) = \frac{-4}{(2x-1)^2}$ ...(ii)

On substituting, x = 1, 2, 3, ..., N in Eq. (ii) and adding, we get

$$g''\left(N+\frac{1}{2}\right) - g''\left(\frac{1}{2}\right) = -4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N-1)^2}\right\}.$$

67 **(d)**

$u + x^2 + y^2, x = s + 3t, y = 2s - t$

Now, $\frac{dx}{ds} = 1, \frac{dy}{ds} = 2$ (1)

$\frac{d^2x}{ds^2} = 0, \frac{d^2y}{ds^2} = 0$ (2)

Now, $u = x^2 + y^2, \frac{du}{ds} = 2x\frac{dx}{ds} + 2y\frac{dy}{ds}$

$$\frac{d^2u}{ds^2} = 2\left(\frac{dx}{ds}\right)^2 + 2x\frac{d^2x}{ds^2} + 2\left(\frac{dy}{ds}\right)^2 + 2y\left(\frac{d^2y}{ds^2}\right)$$

From (1) and (2), $\frac{d^2u}{ds^2} = 2\times 1 + 0 + 2\times 4 + 0 = 10$

68 **(c)**

$f(x) = xe^x$

$f'(x) = e^x + xe^x$

$f''(x) = e^x + e^x + xe^x$

$f'''(x) = 2e^x + e^x + xe^x = 3e^x + xe^x$

...

...

$f^n(x) = ne^x + xe^x$

Now, $f^n(x) = 0$

$\Rightarrow ne^x + xe^x = 0 \Rightarrow x = -n$

69 **(c)**

$y = ae^{mx} + be^{-mx}$

$\Rightarrow \frac{dy}{dx} = ame^{mx} - mbe^{mx}$

Again $\frac{d^2y}{dx^2} = am^2e^{mx} + m^2be^{-mx}$

$\Rightarrow \frac{d^2y}{dx^2} = m^2(ae^{mx} + be^{-mx}) \Rightarrow \frac{d^2y}{dx^2} = m^2y$

$\Rightarrow \frac{d^2y}{dx^2} = m^2y = 0$

70 **(b)**

$y = x + e^x \Rightarrow \frac{dy}{dx} = 1 + e^x \Rightarrow \frac{dx}{dy} = \frac{1}{1+e^x}$

$$\Rightarrow \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{1}{1+e^x}\right) \Rightarrow \frac{d^2x}{dy^2} = \frac{d}{dx}\left(\frac{1}{1+e^x}\right)\frac{dx}{dy}$$

$$\Rightarrow \frac{d^2x}{dy^2} = \frac{-e^x}{(1+e^x)^2}\frac{1}{(1+e^x)} = -\frac{e^x}{(1+e^x)^3}$$

71 **(c)**

$$y = \frac{\log\tan x}{\log\sin x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(\log\sin x)\left(\frac{\sec^2 x}{\tan x}\right) - (\log\tan x)(\cot x)}{(\log\sin x)^2}$$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\pi/4} = \frac{-4}{\log 2}$ (On simplification)

72 **(b)**

Since $g(x)$ is the inverse of function $f(x)$, therefore $gof(x) = I(x)$ for all x

Now $gof(x) = I(x), \forall\, x$

$\Rightarrow (gof)'(x) = 1, \forall\, x$

$\Rightarrow g'(f(x))f'(x) = 1, \forall\, x$

$\Rightarrow g'(f(x)) = \frac{1}{f'(x)}, \forall\, x$

$\Rightarrow g'(f(c)) = \frac{1}{f'(c)}$ (putting $x = c$)

73 **(b)**

$$y = \frac{(a-x)^{3/2} + (x-b)^{3/2}}{\sqrt{a-x} + \sqrt{x-b}}$$

$$= \frac{\left(\sqrt{a-x} + \sqrt{x-b}\right)\left(a - x - \sqrt{a-x}\sqrt{x-b} + \begin{matrix} \\ x-b\end{matrix}\right)}{\sqrt{a-x} + \sqrt{x-b}}$$

$$= a - b - \sqrt{a-x}\sqrt{x-b}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{a-x}}\sqrt{x-6} - \frac{1}{2\sqrt{x-b}}\sqrt{a-x}$$

$$= \frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}}$$

74 **(d)**

Given, $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

On differentiating w.r.t. x, we get

$f'(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} + \begin{vmatrix} x^3 & \sin x & \cos x \\ 0 & 0 & 0 \\ p & p^2 & p^3 \end{vmatrix} + \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$

$\Rightarrow f'(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

$\Rightarrow f''(x) = \begin{vmatrix} 6x & -\sin x & -\cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

and $f'''(x) = \begin{vmatrix} 6 & -\cos x & \sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} + 0 + 0$

$\therefore \quad f'''(0) = \begin{vmatrix} 6 & -1 & 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} = 0$ = independent of p

OR

F(x) = $x^3(-p^3)$ - sinx $(6p^3)$ + cosx $(6p^2+p)$

F'''(x) = $6(-p^3)$ + cosx $(6p^3)$ + sinx$(6p^2+p)$

Hence independent of p

75 **(a)**

Since g is the inverse function of f, we have $f\{g(x)\} = x$

$\Rightarrow \frac{d}{dx}(f\{g(x)\}) = 1$

$\Rightarrow f'\{g(x)\}.g'(x) = 1$

$\Rightarrow \sin\{g(x)\}\, g'(x) = 1$

$\Rightarrow g'(x) = \frac{1}{\sin\{g(x)\}}$

76 **(b)**

$$y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} = \frac{\cos x - \sin x}{\cos x + \sin x} = \frac{1-\tan x}{1+\tan x}$$

$$= \tan\left(\frac{\pi}{4} - x\right)$$

$$\Rightarrow \frac{dy}{dx} = -\sec^2\left(\frac{\pi}{4} - x\right)$$

77 **(a)**

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{f'(x^3)3x^2}{g'(x^2)2x} = \frac{\cos x^3 3x^2}{\sin x^2 2x}$$

$$= \frac{3}{2}x \cos x^3 \operatorname{cosec} x^2$$

78 **(b)**

For $x > 1$, we have $f(x) = |\log|x|| = \log x$

$\Rightarrow f'(x) = \frac{1}{x}$

For $x < -1$, we have $f(x) = |\log|x|| = \log(-x)$

$\Rightarrow f'(x) = \frac{1}{x}$

For $0 < x < 1$, we have $f(x) = |\log|x|| = -\log x$

$\Rightarrow f'(x) = \dfrac{-1}{x}$

For $-1 < x < 0$, we have $f(x) = -\log(-x)$

$\Rightarrow f'(x) = \dfrac{-1}{x}$

Hence, $f'(x) = \begin{cases} \frac{1}{x}, & |x| > 1 \\ -\frac{1}{x}, & |x| < 1 \end{cases}$

79 **(c)**

$f(x) = \sqrt{1 - \sin 2x} = \sqrt{(\cos x - \sin x)^2}$

$= |\cos x - \sin x|$

$= \begin{cases} \cos x - \sin x, & \text{for } 0 \le x \le \pi/4 \\ -(\cos x - \sin x), & \text{for } \pi/4 < x \le \pi/2 \end{cases}$

$= \begin{cases} -(\cos x + \sin x), & \text{for } 0 < x < \pi/4 \\ \cos x + \sin x & \text{for } \pi/4 < x < \pi/2 \end{cases}$

80 **(b)**

$\sqrt{x} = \cos\theta$

$x \in \left(0, \dfrac{1}{2}\right) \Rightarrow \sqrt{x} = \cos\theta \ \in \left(0, \dfrac{1}{\sqrt{2}}\right)$

$\Rightarrow \theta \in \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$

$\Rightarrow 2\theta \in \left(\dfrac{\pi}{2}, \pi\right)$

$\Rightarrow f(x) = 2\sin^{-1}\sqrt{1 - \cos^2\theta} + \sin^{-1}(2\sqrt{\cos^2\theta \sin^2\theta})$

$= 2\sin^{-1}(\sin\theta) + \sin^{-1}(2\sin\theta\cos\theta)$

$= 2\theta + \sin^{-1}(\sin 2\theta)$

$= 2\theta + \pi - 2\theta$

$= \pi$

$\Rightarrow f'(x) = 0$

Integer Answer Type

81 **(5)**

According to question $(a^2 - 2a - 15)e^{ax} + (b^2 - 2b - 15)e^{bx} = 0$

$\Rightarrow (a^2 - 2a - 15) = 0$ and $b^2 - 2b - 15 = 0$

$\Rightarrow (a - 5)(a + 3) = 0$ and $(b - 5)(b + 3) = 0$

$\Rightarrow a = 5$ or -3 and $b = 5$ or -3

$\therefore a \ne b$ hence $a = 5$ and $b = -3$

Or $a = -3$ and $b = 5$

$\Rightarrow ab = -15$

82 **(3)**

$y = \dfrac{x^4 - (x^2 + 2x + 1)}{x^2 - x - 1} = x^2 + x + 1$

$\therefore \dfrac{dy}{dx} = 2x + 1 = ax + b$

Hence $a = 2$ and $b = 1$

83 **(2)**

Since $f(x)$ is odd. Therefore $f(-x) = -f(x) \Rightarrow f'(-x)(-1) = -f'(x)$

$\Rightarrow f'(-x) = f'(x) \therefore f'(-3) = f'(3) = -2$

84 **(5)**

$y = \dfrac{a + bx^{3/2}}{x^{5/4}}$

$\Rightarrow y' \dfrac{\frac{3}{2}bx^{1/2}x^{5/4} - \frac{5}{4}x^{1/4}(a + bx^{3/2})}{x^{5/2}}$

According to the question,

$0 = \dfrac{\frac{3}{2}b5^{1/2}5^{5/4} - \frac{5}{4}5^{1/4}(a + b5^{3/2})}{5^{5/2}}$

$\Rightarrow \dfrac{3b}{2}5^{7/4} - a\dfrac{5^{5/4}}{4} - 5b\dfrac{5^{7/4}}{4} = 0$

$\Rightarrow b5^{7/4} = a5^{5/4}$

$\Rightarrow b\sqrt{5} = a$

$\Rightarrow a : b = \sqrt{5} : 1$

85 **(8)**

$In(f(x)) = In(x - 1) + In(x - 2) + \cdots + In(x - n)$

$\Rightarrow f'(x) = f(x)\left[\dfrac{1}{x - 1} + \dfrac{1}{x - 2} + \cdots + \dfrac{1}{x - n}\right]$

$\Rightarrow f'(x) = (x - 2)(x - 3)(x - n) + (x - 1)(x - 3) \dots (x - n) + \cdots + (x - 1)\ (x - 2) \dots (x - (n - 1))$

$\Rightarrow f'(n) = (n - 1)(n - 2)(n - 3) \cdot 3.2.1$ (all other factors except the last vanishes when $x = n$)

$\Rightarrow 5040 = (n - 1)!$

$\Rightarrow n = 8$